Unified Approach to Prediction of Propagation Over Buildings for All Ranges of Base Station Antenna Height

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Abstract—**This paper presents theoretical results for the dependence on base station antenna height of the average received signal for mobiles at street level. The results apply to residential and commercial sections of cities and to all ranges of antenna height from well above to below that of the surrounding buildings. Assuming all buildings to be of equal heights, the range dependence of the average signal is found by evaluating multiple forward diffraction past rows of buildings. The solution for this diffraction problem for sources near to or below the rooftops gives the dependence of the range index on antenna height and base station height gain, which are in agreement with measurements. These results will be of importance for proposed systems for personal communication services (PCS), which envision the use of base station antennas at the height of lamp posts, as well as cellular mobile radio (CMR).**

I. Introduction

In areas of a city away from the high rise core, base station antennas of height close to that of the surrounding rooftops are of potential importance for cellular mobile radios (CMR's) and for personal communication services (PCS). On the one hand, as the use of cellular telephone grows, lower base station antennas will be employed to achieve cell splitting, even in residential and suburban regions. On the other hand, PCS systems are expected to employ base station antennas at the height of lamp posts [1], [2]. Line of sight (LOS) propagation along the street on which the base station is located will be of great importance in defining the coverage area for low antennas because of the low path loss as compared to propagation over the rooftops. However, coverage areas will extend several blocks on either side of the LOS path, especially in residential areas where the density of users is low. Moreover, interfering signals can be expected to arise as a result of propagation over non-LOS paths. Thus propagation over the rooftops is a factor of concern for both CMR and PCS for coverage and interference prediction.

In this paper we present theoretical expressions for the dependence on base station antenna height of the average received signal for mobiles at street level. To derive these expressions we make use of the viewpoint for predicting the range dependence of the average signal that was developed at Polytechnic [3], [4]. For sections of a city having buildings of relatively uniform height that are closely spaced in rows along streets, the propagation between base station and mobile is viewed as taking place over the rooftops. This propagation path is suggested in Fig. 1 for three rows of buildings between the base station and the mobile. Using results previously obtained for diffraction past multiple obstacles [4], we are able to derive theoretical expressions for path loss that are valid for all base station antenna heights H_S.

The range dependence of the average signal is found by assuming all buildings to be of equal heights h_B [3]. Deviations of the building height from the average, differences in building shape and construction, and the occasional absence of a building from a row lead to variations of the sector average from this range dependence (slow fading) [5]–[7]. For elevated base station antennas, these differences in the building height and construction when taken together lead to the log normal distribution found for the slow fading statistics. In this paper we first provide a systematic way of viewing the sources of path loss, and review its dependence on antenna height for elevated antennas. We show how the results derived here compare with previous results for high base station antennas, and develop simple approximations for antennas well below the rooftops. These results are compared with recently reported measurements [8] for $H_S - h_B$ ranging from -8 to $+6$ m.

II. Contributions to Path Loss

Treating the base station as the source and the mobile as the receiver, the path loss in decibels may be written as the sum of the free space path loss L_0 and the excess loss L_{ex}. Referring to Fig. 1, the free space loss is given by [3]:

$$L_0 = -10\log\left[\left(\frac{\lambda}{4\pi R}\right)^2\right] \tag{1}$$

where λ is the wavelength and R the separation between base station and mobile. The excess path loss can be written as the sum of two parts $L_{\mathrm{ex}} = L_{e1} + L_{e2}$ associated with: 1) the diffraction of the fields at the rooftop before the mobile down the street level; 2) the reduction of the field at the rooftop before the mobile as a result of propagation past the previous rows of buildings.

Assuming the building before the mobile to act as an absorbing screen located at the highest point of the building visible from the mobile, the diffraction down to street level

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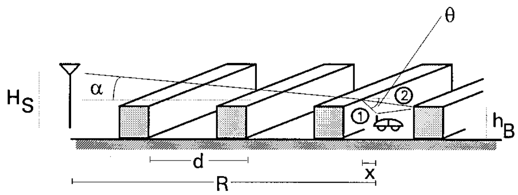

Fig. 1. Radio propagation path in urban environments.

gives the excess loss [9]:

$$L_{e1} = -10\log\left[\frac{G_1(\theta)}{\pi k r}\left(\frac{1}{\theta} - \frac{1}{2\pi + \theta}\right)^2\right] \quad (2)$$

where $G_1(\theta)$ is the gain of the mobile antenna in the direction θ indicated in Fig. 1 and given by

$$\theta = \tan^{-1}\left(\frac{h_B - h_m}{x}\right) \quad (3)$$

and

$$r = \sqrt{(h_B - h_m)^2 + x^2}. \quad (4)$$

Here h_B is the average height of the building, h_m is the height of the mobile antenna, and x is the horizontal distance between the mobile and the diffracting edges of the building closest to the mobile. A factor of 2 has been included in (2) to represent the influence on the average signal of indirect paths, such as ② in Fig. 1, as well as the direct path ①. Because of the deep Rayleigh fading that is observed, the fields arriving on direct and indirect paths must be of nearly equal amplitudes. Thus when the received signal is averaged over a distance of 10 to 20 wavelengths, the resulting average signal (proportional to the field squared) will be twice that of the direct path. Height gain of the mobile antenna is seen to be given by L_{e1}. Previous studies have shown that the height gain given by (2) is in good agreement with measurements [10].

Path loss L_{e2} due to the gain of the base station antenna G_2 and to the reduction by intervening buildings of the field reaching the rooftop before the mobile is expressed as

$$L_{e2} = -10\log(G_2 Q^2). \quad (5)$$

The gain G_2 is that in the direction to the highest building edge visible from the base station antenna when it is below the surrounding rooftops, or the gain in the horizontal plane when the base station antenna is above the surrounding rooftops. Evaluation of Q has been the subject of previous studies [3], [4]. Because forward diffraction through small angles is not sensitive to obstacle shape, in these studies the rows of buildings were replaced by absorbing screens for simplicity.

III. L_{e2} for Elevated Antennas

For elevated antennas and when the signal traverses many rows of buildings, it was found that the dependence of Q on base station antenna height H_S and row spacing d was through the dimensionless parameter [3]

$$g_p = \alpha\sqrt{\frac{d}{\lambda}} \quad (6)$$

where

$$\alpha = \tan^{-1}\frac{H_S - h_B}{R} \approx \frac{H_S - h_B}{R}. \quad (7)$$

The dependence $Q(g_p)$ was computed numerically. A simple fit to the computed results that is accurate to within 0.8 dB over the range $0.01 < g_p < 0.4$ was found to be [3]:

$$Q(g_p) = 2.35 g_p^{0.9}. \quad (8)$$

For 900 MHz signals, a typical row spacing of $d = 40$ m, and $H_S - h_B = 10$ m, the foregoing range of g_p correspond to values of R in the range $0.3 < R < 11$ km, which is typical of the macrocells of CMR systems. Note from (6) and (7) that Q varies as $1/R^{0.9}$, so that combining the free space path loss and Q one finds that the average received signal varies as $1/R^{3.8}$. Also, the average received signal is seen from (6)–(8) to have dependence on base station antenna height that is proportional to $(H_S - h_B)^{1.8}$.

In order to apply the theory for smaller values of R, a polynomial was fit to the numerical results. With an accuracy greater than 0.5 dB over the range $0.01 < g_p < 1$, such a fit gives

$$Q(g_p) = 3.502 g_p - 3.327 g_p^2 + 0.962 g_p^3 \qquad 0.01 < g_p < 1. \quad (9)$$

Since the leading term is linear in g_p, for large R or small $H_S - h_B$ (small values of g_p) the variation is close to that of (8). However, as R decreases or $H_S - h_B$ increases, g_p approaches unity and Q departs from a linear variation. This variation was found to describe measurements made in Denmark for small R [10].

IV. L_{e2} for Low Antennas

When finding the effect of M intervening rows of buildings for low antennas that may be near, or even below the rooftops of the surrounding building, it is important to consider the cases of small M for coverage prediction and large M for interference prediction. Again approximating the rows of buildings by absorbing screens, Xia and Bertoni [4] were able to evaluate this effect for values of M ranging to 100 or more considering the two dimensional canonical problem shown in Fig. 2. Here a line source is located at point P_1, a distance d before a series of $M - 1$ absorbing screens spaced d apart. The field reaching a point P_2 that is at the top of the last screen, which correspond to the field incident on the rooftop of the last row of buildings before the mobile, was then evaluated. The factor Q_M giving the reduction of the line source field at P_2 due to the screens is the same that would be found for a point source at P_1. This is because the spreading of the point source fields out of the plane of the drawing is the same with or without the presence of the screens.

Xia and Bertoni were able to express the factor Q_M in terms of Boersma functions $I_{n,q}$ as [4], [11]:

$$Q_M = \sqrt{M}\left|\sum_{q=0}^{\infty}\frac{1}{q!}\left[2g_c\sqrt{j\pi}\right]^q I_{M-1,q}\right| \quad (10)$$

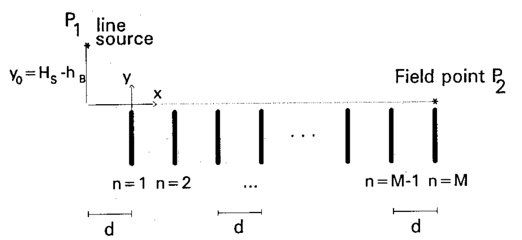

Fig. 2. Using absorbing half planes instead of buildings.

where the dimensionless parameter g_c is given by

$$g_c = y_0 \frac{1}{\sqrt{\lambda d}} \tag{11}$$

and

$$y_0 = H_S - h_B. \tag{12}$$

The Boersma functions are found from the recursion relation

$$I_{M-1,q} = \frac{(M-1)(q-1)}{2M} I_{M-1,q-2} + \frac{1}{2\sqrt{\pi}M} \sum_{n=1}^{M-2} \frac{I_{M-1,q-1}}{(M-1-n)^{1/2}} \tag{13}$$

with initials terms

$$I_{M-1,0} = \frac{1}{M^{3/2}} \tag{14}$$

$$I_{M-1,1} = \frac{1}{4\sqrt{\pi}} \sum_{n=0}^{M-1} \frac{1}{n^{3/2}(M-n)^{3/2}}. \tag{15}$$

From these expressions it is seen that Q_M depends on the source height y_0 above or below the rooftops and the row separation d only through the parameter g_c of (11). For g_c small, the series in (10) for Q_M converges rapidly. Using the foregoing expressions we have computed Q_M for different values of y_0 at 900 MHz and 1800 MHz, assuming a row spacing d of 50 m. The results of these calculations are shown in Figs. 3 and 4 for y_0 varying in steps of 1.25 m.

To help in the understanding of these curves observe that for $y_0 = 0$ meters or $g_c = 0$ that $Q_M(0) = 1/M$, so that $\log Q_M$ decreases linearly with log of distance from the base station. For antennas below the rooftops ($y_0 < 0$), it is seen from the slope of the curves in Fig. 3 and 4 that Q_M initially decreases more rapidly than $1/M$, but quickly approaches the $1/M$ variation. Conversely, antennas above the rooftops ($y_0 > 0$) initially give a rate of decrease for Q_M that is less than $1/M$, eventually achieving the $1/M$ variation for large M.

The foregoing discussion is made quantitative in Figs. 5 and 6, where we have plotted the negative of the slopes of the curves in Figs. 3 and 4, as found from the logarithmic derivative:

$$s = -\frac{\log(Q_{M+1}/Q_M)}{\log[(M+1)/M]}. \tag{16}$$

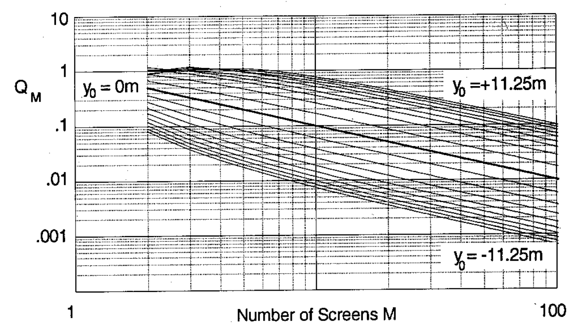

Fig. 3. Field after multiple diffraction over absorbing screens. Frequency of 900 MHz and $d = 50$ m.

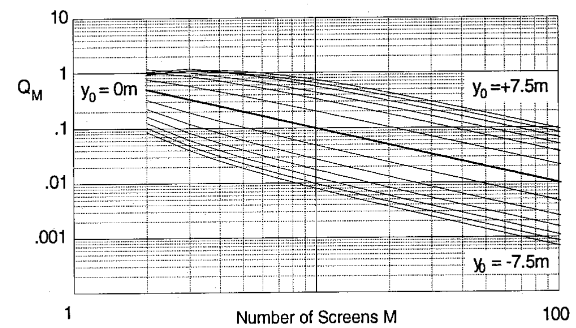

Fig. 4. Field after multiple diffraction over absorbing screens. Frequency of 1800 MHz and $d = 50$ m.

For $y_0 = 0$ the slope is $s = +1$, while for $y_0 < 0$ it quickly approaches this value. On the other hand, for $y_0 > 0$ the slope approaches +1 in a much more gradual manner. High antennas show a slope closer to 0.9 for M in the range of 20 to 60, in accordance with the previous discussion. It should be noted that the index n in the range dependence $1/R^n$ of the received signal is equal to the sum of the free space index and twice the slope s in Figs. 5 and 6, so that $n = 2 + 2s$. The value of n is plotted versus y_0 in Figs. 7 and 8 for $M = 10$. It is seen that the range index starts at a value greater than 4 and decreases to a value less than 4 as the antenna height varies from below to above the rooftops.

Recent measurements made in the Mission and Sunset Districts of San Francisco for low base station antennas were processed to obtain the range index for propagation paths passing over the rooftops [8]. The index was taken from the slope of the regression fit to path loss measurements for distances ranging from 100 m to 1.2 km. When expressed in terms of $\log R$, this measurement range is approximately centered on a distance of 500 m, which corresponds to $M = 10$ for $d = 50$ m. Thus the slopes obtained from the measurements at 901 MHz and 1937 MHz can be meaningfully compared to the computed slopes for $M = 10$. The range of slopes obtained on different measurement paths are shown as dark vertical bars in Figs. 7 and 8. It is seen that the theoretical model correctly predicts the trend of the variation of n with antenna height.

V. Base Station Height Gain

The excess loss term L_{e2} of (5) gives the base station antenna height gain, as well as the range dependence of the

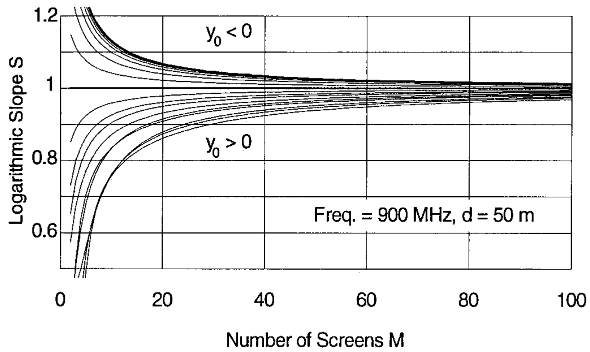

Fig. 5. Slope of the field after multiple diffraction over absorbing screens.

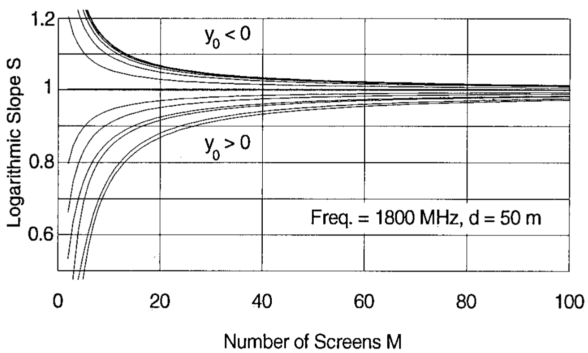

Fig. 6. Slope of the field after multiple diffraction over absorbing screens.

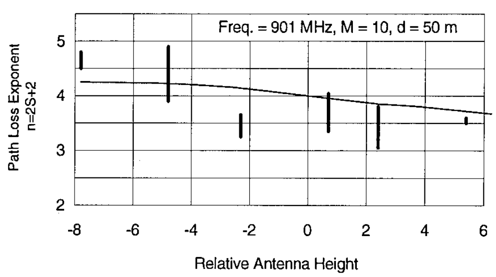

Fig. 7. Slope of the field after multiple diffraction versus $y_0 = H_S - h_B$.

excess path loss. for larger values of M, where the slope s approaches unity, it is possible to separate Q_M into two factors, one giving the height gain and the other the range dependence. Recognizing from Fig. 1 that $M = (R - x)/d \approx R/d$, the separation takes the form

$$Q_M = \left(\frac{d}{R}\right) G_H(y_0). \tag{17}$$

For small values of M, such a separation is not possible, and the height gain will have some dependence on M. In Fig. 9 we have plotted the variation of Q_M with y_0 for $M = 20$ at 900 MHz. For comparison we have also plotted $Q(g_p)$ from (8), which is valid for high antennas. It is seen that for $y_0 > \sqrt{\lambda d}$, the simple high antenna solution gives a good approximation to Q_M.

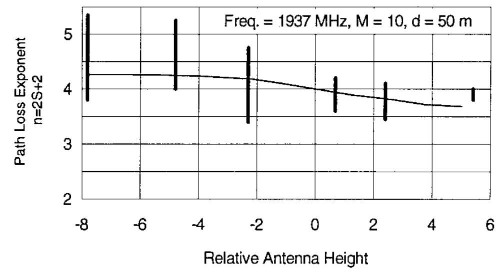

Fig. 8. Slope of the field after multiple diffraction versus $y_0 = H_S - h_B$.

A simple alternate expression for Q_M can be derived when the base station antenna is sufficiently below the rooftops so that the second row of buildings lies outside the transition region of the first row, ($|y_0| > \sqrt{\lambda d}$). In this case, the edge of the first screen acts approximately as a line source for the remaining $M-1$ screens, as suggested in Fig. 2. The amplitude of this equivalent line source depends on the amplitude of the incident cylindrical wave and the diffraction coefficient for the angle $\varphi = -\tan^{-1}[(H_S - h_B)/d]$ in Fig. 2. This approximation leads to the expression

$$Q_M = \left(\frac{d}{R-d}\right)\left[\frac{1}{\sqrt{2\pi k \rho}}\left(\frac{1}{\varphi} - \frac{1}{2\pi + \varphi}\right)\right]. \tag{18}$$

where

$$\rho = \sqrt{(H_S - h_B)^2 + d^2}. \tag{19}$$

For $R \gg d$, the first factor in (18) gives the $1/R$ dependence obtained when the base station antenna is a rooftop level. The second factor gives the antenna height gain for antennas below rooftop level. In Fig. 9 we have plotted the dependence on y_0 of the simplified expression (18). For $y_0 < -\sqrt{\lambda d}$, (18) is seen to correctly describe the variation of Q_M, but is about 2 dB lower.

The regression fits to the measurements made in residential and commercial sections of San Francisco for propagation directly over rows of buildings can also be used to validate the predictions made here. Since the measurements were made in two different sections of San Francisco with different building heights, we have extracted from them the loss L_{e2} for comparison. Starting with the 1-km intercept to the regression fit, we remove the free space path loss L_0 and the diffraction loss L_{e1} describing propagation from the rooftop nearest the mobile down to its level. Since the antenna gain patterns in the vertical plane are not reported, we have assumed that $G_1 = G_2 = 1$, so that the remaining loss can then be compared to $-20\log(Q)$. In Figs. 10 and 11 we have plotted the resulting values for 901 MHz and 1937 MHz as open squares for the various values $H_S - h_B$ used in the measurements. For comparison we have plotted $-20\log(Q)$. It is seen that there is a good agreement between theory and measurement at 901 MHz both for the absolute value and its variation with antenna height. At 1937 MHz the agreement is not as good for low base station antennas, but the trend in both measurements and theory are the same.

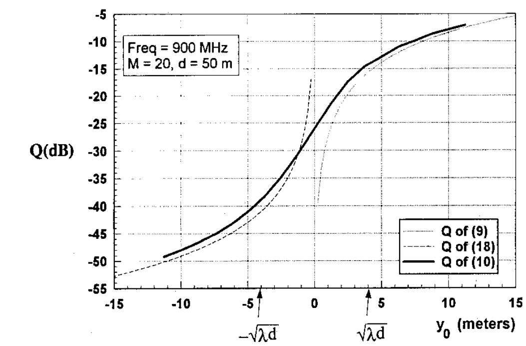

Fig. 9. Comparison among models inside and outside the transition region.

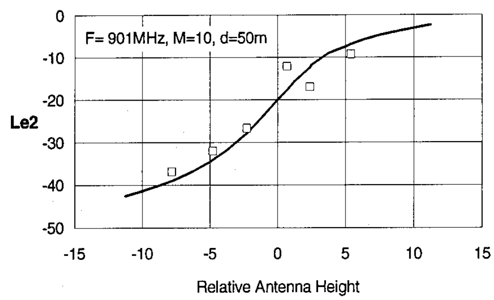

Fig. 10. Antenna height gain and measurements versus $y_0 = H_S - h_B$ for 901 MHz.

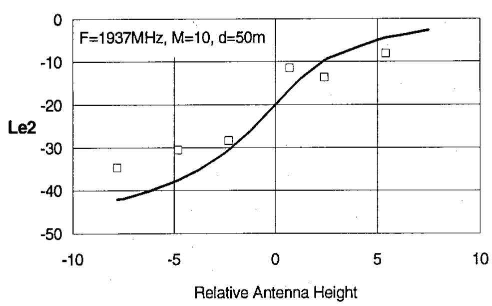

Fig. 11. Antenna height gain and measurements versus $y_0 = H_S - h_B$ for 1937 MHz.

VI. Conclusion

A theoretical model has been developed to predict the propagation over rows of buildings in residential and commercial sections of cities for the low base station antennas envisioned for PCS systems. The theory predicts the distance dependence of the average signal, as well as the height gain of the base station antenna. Theoretical predictions are in agreement with reported measurements.

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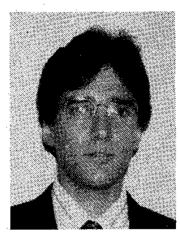

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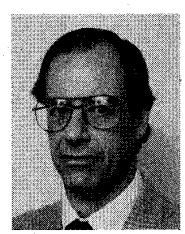

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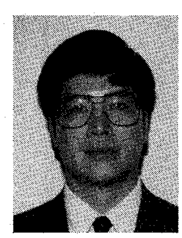

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